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Function of two or more Variables:—

Let x, y be two independent variables of a variable z takes a value corresponding to a pair of values (x, y) then we say z is a function of two variables x, y and

$$\text{we write } z = f(x, y)$$

e.g. $z = x^2 + xy$ is a function two variable

The above definition can be extended for more than two variables.

Partial differentiation:—

A partial differentiation of a function of two or more variables is the ordinary differentiation w.r to one of the variable when all the remaining variables are kept constants

Let $u = u(x, y)$ be a function two variables

hence u is dependent variables
& x, y are independent.

The partial differentiation of $u = u(x, y)$ w.r to x is the ordinary derivative of u w.r to x . Keeping y is constant &

it is denoted by

First order partial derivative of u w.r to x and is defined as

$$\frac{\partial u}{\partial x} = \lim_{\Delta x \rightarrow 0} \frac{u(x+\Delta x, y) - u(x, y)}{\Delta x}$$

Similarly partial derivative of u w.r to y is the ordinary derivative of u w.r to y keeping x is constant is denoted

defined by

$$\frac{\partial u}{\partial y} = \lim_{\Delta y \rightarrow 0} \frac{u(x, y+\Delta y) - u(x, y)}{\Delta y} \quad \text{if } \frac{\partial u}{\partial y}$$

Similarly $\frac{\partial^2 u}{\partial x^2}$, $\frac{\partial^2 u}{\partial y^2}$, $\frac{\partial^2 u}{\partial x \partial x}$ are known.

as second order partial derivatives of w.r to x, y defined by

$$\begin{aligned} \frac{\partial^2 u}{\partial x^2} &= \frac{\partial}{\partial x} \left(\frac{\partial u}{\partial x} \right) \\ &= \lim_{\Delta x \rightarrow 0} \frac{u_x(x+\Delta x, y) - u_x(x, y)}{\Delta x} \end{aligned}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial y} \right) = \lim_{\Delta y \rightarrow 0} \frac{u_y(x, y+\Delta y) - u_y(x, y)}{\Delta y}$$

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial}{\partial x} \left(\frac{\partial u}{\partial y} \right)$$